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Invariant Submanifold of $\tilde{\psi}(k,k-1.....1)$ Structure Manifold

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Abstract

In this paper, we have studied various properties of a $\tilde{\psi}(k,k-1.....1)$ structure manifold and its invariant submanifold, where *k* is positive integer greater than 3. Under two different assumptions, the nature of induced structure ψ, has also been discussed.

Keywords: Invariant submanifold, Nijenhuis tensor, projection operators and complementary distributions.

Introduction

Let V^m be a C^∞ m-dimensional Riemannian manifold imbedded in a C^∞ n-dimensional Riemannian manifold M^n, where $m<n$. The imbedding being denoted by

$$f: V^m \longrightarrow M^n$$

Let B be the mapping induced by *f* i.e. *B=df*

$$df: T(V) \longrightarrow T(M)$$

Let *T* (*V,M*) be the set of all vectors tangent to the submanifold *f*(*V*). It iswell known that

$$B: T(V) \longrightarrow T(V,M)$$

Is an isomorphism. The set of all vectors normal to $f(V)$ forms a vector bundle over $f(V)$, which we shall denote by $N(V,M)$. We call $N(V,M)$ the normal bundle of V^m. The vector bundle induced by f from $N(V,M)$ is denoted by $N(V)$. We denote by $C: N(V) \longrightarrow N(V,M)$ the natural isomorphism and by $\eta_s^r(V)$ the space of all C^∞ tensor fields of type (r,s) associated with N (V).

Thus $\zeta_0^0(V) = \eta_0^0(V)$ is the space of all C^∞ functions defined on V^m while an element of $\eta_0^1(V)$ is a C^∞ vector field normal to V^m and an element of $\zeta_0^1(V)$ is a C^∞ vector field tangential to V^m.

Let $\bar{X}$ and $\bar{Y}$ be vector fields defined along $f(V)$ and $\tilde{X}, \tilde{Y}$ be the local extensions of $\bar{X}$ and $\bar{Y}$ respectively. Then $[\tilde{X}, \tilde{Y}]$ is a vector field tangential to M^n and its restriction $[\tilde{X}, \tilde{Y}]/f(V)$ to $f(V)$ is determined independently of the choice of these local extension $\tilde{X}$ and $\tilde{Y}$. Thus $[\bar{X}, \bar{Y}]$ is defined as

$$[\bar{X}, \bar{Y}] = [\tilde{X}, \tilde{Y}]/f(V) \qquad (1.1)$$

Since B is an isomorphism

$$[BX, BY] = B[X,Y] \quad \text{for all} \quad X, Y \in \zeta_0^1(V) \qquad (1.2)$$

Let $\bar{G}$ be the Riemannain metric tensor of M^n, we define *g* and *g** on V^m and *N* (*V*) respectively as $g(X_1, X_2) = \tilde{G}(BX_1, BX_2)\, f,$ (1.3) and $g^*(N_1, N_2) = \tilde{G}(CN_1, CN_2)$ (1.4)

For all $X_1, X_2 \in \zeta_0^1(V)$ and $N_1, N_2 \in \eta_0^1(V)$

It can be verified that *g* and g^* are the induced metrics on V^m and *N* (*V*) respectively.

Let $\tilde{\nabla}$ be the Riemannian connection determined by $\tilde{G}$ in M^n, then $\tilde{\nabla}$ induces a connection ∇ in $f(V)$ defined by

$$\nabla_{\bar{X}}\bar{Y} = \tilde{\nabla}_{\tilde{X}}\tilde{Y} / f(V) \tag{1.5}$$

where $\bar{X}$ and $\bar{Y}$ are arbitrary C^∞ vector fields defined along $f(V)$ and tangential to $f(V)$.

Let us suppose that M^n is a $C^\infty\ \tilde{\psi}(k, k-1.....1)$ structure manifold with structure tensor $\tilde{\psi}$ of type (1,1) satisfying

$$\tilde{\psi}^k + \tilde{\psi}^{k-1} + ... + \tilde{\psi} = 0 \tag{1.6}$$

Let $\tilde{L}$ and $\tilde{M}$ be the complementary distributions corresponding to the projection operators

$$\tilde{l} = \tilde{\psi}^k, \qquad \tilde{m} = I - \tilde{\psi}^k \tag{1.7}$$

where I denotes the identity operator.

From (1.6) and (1.7), we have

(a) $\tilde{l} + \tilde{m} = I$

(b) $\tilde{l}^2 = \tilde{l}$

(c) $\tilde{m}^2 = \tilde{m}$

(d) $\tilde{l}\ \tilde{m} = \tilde{m}\ \tilde{l} = 0$ (1.8)

Let D_l and D_m be the subspaces inherited by complementary projection operators l and m respectively.

We define

$$D_l = \{X \in T_p(V) : lX = X, mX = 0\}$$

$$D_m = \{X \in T_p(V) : mX = X, lX = 0\}$$

Thus $T_p(V) = D_l + D_m$

Also $Ker\ l = \{X : lX = 0\} = D_m$

$Ker\ m = \{X : mX = 0\} = D_l$

at each point p of $f(V)$.

Invariant Submanifold of $\tilde{\psi}(k, k-1.....1)$ Structure Manifold

We call V^m to be invariant submanifold of M^n if the tangent space $T^p(f(V))$ of $f(V)$ is invariant by the linear mapping $\tilde{\psi}$ at each point *p* of $f(V)$. Thus

$$\tilde{\psi}BX = B\psi X, \text{ for all } X \in \zeta_0^1(V), \tag{2.1}$$

and ψ being a (1,1) tensor field in V^m.

Theorem 2.1

Let $\tilde{N}$ and *N* be the Nijenhuis tensors determined by $\tilde{\psi}$ and ψ in M^n and V^m respectively, then

$$\tilde{N}(BX, BY) = BN(X, Y), \text{ for } X, Y \in \zeta_0^1(V) \tag{2.2}$$

Proof We have, by using (1.2) and (2.1) (2.3)

$$\begin{aligned} \tilde{N}(BX, BY) &= [\tilde{\psi}BX, \tilde{\psi}BY] + \tilde{\psi}^2[BX, BY] \\ &\quad - \tilde{\psi}[\tilde{\psi}BX, BY] - \tilde{\psi}[BX, \tilde{\psi}BY] \\ &= [B\psi X, B\psi Y] + \tilde{\psi}^2 B[X, Y] \\ &\quad - \tilde{\psi}[B\psi X, BY] - \tilde{\psi}[BX, B\psi Y] \\ &= B[\psi X, \psi Y] + B\psi^2[X, Y] - \tilde{\psi}B[\psi X, Y] \\ &\quad - \tilde{\psi}B[X, \psi Y] \\ &= B\{[\psi X, \psi Y] + \psi^2[X, Y] - \psi[\psi X, Y] \\ &\quad - \psi[X, \psi Y]\} \\ &= B\,N(X, Y) \end{aligned}$$

Distribution $\tilde{M}$ Never Being Tangential To $f(V)$

Theorem 3.1

If the distribution $\tilde{M}$ is never tangential to $f(V)$, then

$\tilde{m}(BX) = 0$ (3.1) for all $X \in \zeta_0^1(V)$

and the induced structure ψ on V^m satisfies

$$\psi^{k-1} + \psi^{k-2} + ... + \psi + I = 0 \tag{3.2}$$

Thus ψ is $(k-1, k-2, ...0)$.

Proof

if possible $\tilde{m}(BX) \neq 0$. From (2.1) We get

$\tilde{\psi}^\alpha BX = B\psi^\alpha X$; $1 \le \alpha \le$ k-1 from (1.7) and (3.3)

$$\begin{aligned} \tilde{m}(BX) &= (I - \tilde{\psi}^k) BX \\ &= (I + \tilde{\psi} + \tilde{\psi}^2 + ... + \tilde{\psi}^{k-1}) BX \\ &= BX + B\psi X + B\psi^2 X + ... + B\psi^{k-1} X \end{aligned}$$

$$\tilde{m}(BX) = B(X + \psi X + \psi^2 X + ... + \psi^{k-1} X) \tag{3.4}$$

This relation shows that $\tilde{m}(BX)$ is tangential to $f(V)$ which contradicts the hypothesis. Thus $\tilde{m}(BX)$ = 0. Using this result in (3.4) and remembering that *B* is an isomorphism, We get

$$\psi^{k-1} + \psi^{k-2} + ... + \psi + I = 0 \tag{3.5}$$

Theorem 3.2

Let $\tilde{M}$ be never tangential to $f(V)$, then

$$\underset{\tilde{m}}{\tilde{N}}(BX, BY) = 0 \tag{3.6}$$

Proof

We have

$$\underset{\tilde m}{\tilde N}(BX, BY) = [\tilde m BX, \tilde m BY] + \tilde m^2 [BX, BY]$$
$$- \tilde m [\tilde m BX, BY] - \tilde m [BX, \tilde m BY] \quad (3.7)$$

Using (1.2), (1.8) (c) and (3.1), we get (3.6).

Theorem 3.3

Let $\tilde M$ be never tangential to $f(V)$, then

$$\underset{\tilde l}{\tilde N}(BX, BY) = 0 \quad (3.8)$$

Proof

We have

$$\underset{\tilde l}{\tilde N}(BX, BY) = [\tilde l BX, \tilde l BY] + \tilde l^2 [BX, BY] - \tilde l [\tilde l BX, BY]$$
$$- \tilde l [BX, \tilde l BY] \quad (3.9)$$

Theoren 3.4

Let $\tilde M$ be never tangential to $f(V)$. Define

$$\tilde H(\tilde X, \tilde Y) = \tilde N(\tilde X, \tilde Y) - \tilde N(\tilde m\tilde X, \tilde Y) - \tilde N(\tilde X, \tilde m\tilde Y)$$
$$+ \tilde N(\tilde m\tilde X, \tilde m\tilde Y) \quad (3.10)$$

For all $\tilde X, \tilde Y \in \zeta_0^1(M)$, then

$$\tilde H(BX, BY) = BN(X, Y)$$

Proof

Using $\tilde X = BX$, $\tilde Y = BY$ and (2.2), (3.1) in (3.10) We get (3.11).

Distribution $\tilde M$ Always Being Tangential to $f(V)$

Theorem 4.1

Let $\tilde M$ Be Always Tangential To $f(V)$,

Then (a) $\tilde m(BX) = Bm X$

(b) $\tilde l(BX) = Bl X$

Proof

From (3.4), We get (4.1) (a). Also

$$l = \psi^k$$
$$lX = \psi^k X \quad (4.2)$$
$$BlX = B\psi^k X$$

Using (2.1) in (4.3)

$$BlX = \tilde\psi^k BX = \tilde l(BX), \quad (4.4)$$

which is (4.1) (b).

Theorem 4.2

Let $\tilde M$ be always tangential to $f(V)$, then *l* and *m* satisfy

(a) l +m =I (b) lm= ml =0 (c) l^2=l (d) m^2 =m. (4.5)

Proof

Using (1.8) and (4.1) We get the results.

Theorem 4.3

If $\tilde M$ is always tangential to $f(V)$, then

$$\psi^k + \psi^{k-1} + ... + \psi = 0 \quad (4.6)$$

Proof

From (2.1)

$$\tilde\psi^k BX = B\psi^k X \quad (4.7)$$

Using (1.6) in (4.7)

$$(-\tilde\psi - \tilde\psi^2 - ..... - \tilde\psi^{k-1}) BX = B\psi^k X$$
$$-(B\psi X + B\psi^2 X + ... + B\psi^{k-1} X) = B\psi^k X$$

Or $\psi^k + \psi^{k-1} + ... + \psi = 0$ which is (4.6)

Theorem 4.4

If $\tilde M$ Is always tangential to $f(V)$ then as in (3.10)

$$\tilde H(BX, BY) = BH(X, Y) \quad (4.8)$$

Proof

From (3.10) we get

$$\tilde H(BX, BY) = \tilde N(BX, BY) - \tilde N(\tilde m BX, BY)$$
$$- \tilde N(BX, \tilde m BY) + \tilde N(\tilde m BX, \tilde m BY) \quad (4.9)$$

Using (4.1) (a) and (2.2) in (4.9) we get (4.8).

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